5 Arithmetic Progressions

Fastrack Revision

- Some numbers arranged in a definite order according to a definite rule, are said to form a sequence, where each number is called a term.
- A sequence whose terms follow a certain rule, is called a progression.
- A sequence in which each term differs from its preceding term by same constant except the first term is called an Arithmetic Progression (AP). This constant is called the common difference (d) of the AP. It can be positive, negative or zero.
- The general form of an AP is: $a, a+d, a+2d, \ldots$ where a is the first term.
- If a, b, c are three terms in AP, then $b - a = c - b$, i.e., $2b = a + c$.
- An AP with finite number of terms is a finite AP and which does not have finite number of terms is an infinite AP. Infinite AP's do not have a last term.
- In an AP, if we add, subtract, multiply or divide each term by the same non-zero number, then the resulting sequence would always be an AP.

Knowledge BOOSTER

1. *Three terms in AP should be taken as: $a-d, a, a+d$.*
2. *Four terms in AP should be taken as: $a-3d, a-d, a+d, a+3d$.*
3. *Five terms in AP should be taken as: $a-2d, a-d, a, a+d, a+2d$.*

- **General Term of an AP:** If the first term of an AP is a, common difference is d and its last term is l, then its nth or general term is given by $T_n = a_n = l = a + (n-1)d$.
- If nth term of an AP is a_n, then the common difference is determined by $d = a_n - a_{n-1}$.
- **nth Term from the End of an AP:** If the first term of an AP is a, its common difference is d and its last term is l, then nth term from the end $= l - (n-1)d$.

  And pth term from the end $= (n-p+1)$th term from the beginning.
- **Sum of n Terms of an AP:** If S_n is the sum of n terms of an AP, then $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}[a + l]$

  where a is the first term and l is the last term.

Knowledge BOOSTER

1. *If sum of n terms (S_n) of an AP is given, then nth term (a_n) of an AP can be determined by $a_n = S_n - S_{n-1}$ and common difference $d = a_n - a_{n-1} = S_n - 2S_{n-1} + S_{n-2}$.*
2. **Arithmetic Mean (AM) between Two Numbers:** *If A is the AM between a and b, then*

$$A = \frac{a+b}{2}$$

Practice Exercise

Multiple Choice Questions

Q 1. 5th term of the sequence, whose nth term is $4n + 2$, is:

a. 20 b. 22 c. 18 d. 23

Q 2. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ is:

a. 1 b. $1/p$ c. -1 d. $-1/p$

Q 3. If $k+2$, $4k-6$ and $3k-2$ are three consecutive terms of an AP, then the value of k is: [CBSE 2023]

a. 3 b. -3 c. -4 d. 4

Q 4. The first four terms of an AP, whose first term is -2 and the common difference is -2, are: [NCERT EXEMPLAR]

a. $-2, 0, 2, 4$ b. $-2, 4, -8, 16$ c. $-2, -4, -6, -8$ d. $-2, -4, -8, -16$

Q 5. The next two terms of an AP sequence 5, 8, 11, ... are:

a. 13, 16 b. 14, 17 c. 15, 18 d. 12, 15

Q 6. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then the value of a is: [CBSE 2023]

a. 6 b. 7 c. 20 d. 28

Q 7. The seventh term of an AP whose first term is 28 and common difference -4 is: [CBSE 2023]

a. 0 b. 4 c. 52 d. 56

Q 8. How many terms are there in the AP given below?

14, 19, 24, 29,......, 119 *[CBSE 2023]*

a. 18 b. 14

c. 22 d. 21

Q 9. The nth term of the AP: $a, 3a, 5a, \ldots$ is: *[CBSE 2020]*

a. na b. $(2n-1)\,a$

c. $(2n+1)\,a$ d. $2na$

Q 10. Which term of the AP: 21, 42, 63, 84,... is 210? *[NCERT EXEMPLAR]*

a. 9th b. 10th

c. 11th d. 12th

Q 11. The 21st term of the AP, whose first two terms are −3 and 4, is: *[NCERT EXEMPLAR]*

a. 17 b. 137

c. 143 d. −143

Q 12. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$? *[NCERT EXEMPLAR]*

a. 5 b. 20 c. 25 d. 30

Q 13. The 7th term from the end of the AP:

17, 14, 11,......, − 40 is:

a. − 18 b. − 22 c. − 25 d. − 20

Q 14. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term? *[NCERT EXEMPLAR]*

a. 30 b. 33 c. 37 d. 38

Q 15. If the sum of the first n terms of an AP be $3n^2 + n$ and its common difference is 6, then its first term is: *[CBSE 2023]*

a. 2 b. 3 c. 1 d. 4

Q 16. The sum of the first 21 terms of an AP :

16, 12, 8, 4,...... is: *[CBSE 2023]*

a. −480 b. −504

c. 1176 d. − 484

Q 17. In an AP, if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is equal to:

a. 38 b. 39 c. 40 d. 41

Q 18. The number of terms of an AP:

64, 60, 56,...., whose sum is 544, is:

a. 15, 16 b. 16, 17 c. 14, 15 d. 17, 18

Assertion & Reason Type Questions

Directions (Q. Nos. 19-23): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 19. Assertion (A): $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$ is in Arithmetic Progression.

Reason (R): The terms of an Arithmetic Progression cannot have both positive and negative rational numbers. *[CBSE SQP 2023-24]*

Q 20. Assertion (A): The nth term of the sequence

$-8, -4, 0, 4, \ldots$ is $4n - 12$.

Reason (R): The nth term of an AP is determined by $T_n = a + (n-1)d$.

Q 21. Assertion (A): The common difference of an AP in which $a_{20} - a_{16} = 20$ is 5

Reason (R): The nth term of the sequence $\sqrt{2}, \sqrt{4}, \sqrt{18}, \ldots$ is $\sqrt{2}\,n$.

Q 22. Assertion (A): a, b, c are in AP if and only if $2b = a + c$.

Reason (R): The sum of first n odd natural numbers is n^2. *[CBSE 2023]*

Q 23. Assertion (A): If sum of first n terms of an AP is $S_n = 6n^2 - 2n$, then nth term of an AP is $12n - 8$.

Reason (R): Suppose S_n be the sum of n terms of an AP, then nth term of an AP is $T_n = S_{n-1} - S_n$.

Fill in the Blanks Type Questions

Q 24. If the common difference is ...., then each term of the AP will be same as the first term of the AP.

Q 25. The value of k for which $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP, is .... *[NCERT EXEMPLAR]*

Q 26. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be ....

Q 27. .... term of the AP: 24, 21, 18, 15, ... is the first negative term.

Q 28. The sum of 10 terms of an AP: − 8, − 6, − 4, ... is ....

True/False Type Questions

Q 29. If we multiply each term of an AP by 2, then the resulting sequence is an AP.

Q 30. In an AP, if $a = 1$, $a_n = 20$ and $n = 38$, then sum of first 38 terms is 499.

Q 31. If the first term of an AP is −5 and the common difference is 2, then the sum of the first 6 terms is 5.

Q 32. 20 terms of AP: 18, 16, 14, ... should be taken so that their sum is zero.

Q 33. If sum of the first n terms of an AP is given by $S_n = 3n^2 + 4$, then its nth term is $6n - 3$.

Solutions

1. (b) Given, $a_n = 4n + 2$

Put $n = 5$, we get

$$a_5 = 4(5) + 2 = 20 + 2 = 22$$

2. (c) Common difference

$$= \frac{1-p}{p} - \frac{1}{p}$$

$$= \frac{1}{p} - 1 - \frac{1}{p} = -1$$

3. (a) Given terms of an AP are

$$k + 2, 4k - 6 \text{ and } 3k - 2.$$

Therefore, common difference of each term should be equal.

So, $(4k - 6) - (k + 2) = (3k - 2) - (4k - 6)$

$\Rightarrow \quad 3k - 8 = -k + 4$

$\Rightarrow \quad 4k = 12 \Rightarrow k = 3$

4. (c) Let a be the first term and d be the common difference of an AP.

Then $\quad a = -2$ and $d = -2$

First four terms of an AP are

$$a, a + d, a + 2d, a + 3d.$$

Here, $\quad a = -2$

$$a + d = -2 - 2 = -4$$

$$a + 2d = -2 + 2(-2) = -2 - 4 = -6$$

and $\quad a + 3d = -2 + 3(-2) = -2 - 6 = -8$

Hence, first four terms of an AP are $-2, -4, -6, -8$.

5. (b) Given AP : 5, 8, 11, ...

Here, $\quad a_1 = 5$, $a_2 = 8$ and $a_3 = 11$

$\therefore$ Common difference $(d) = a_2 - a_1 = 8 - 5 = 3$

So, next two terms *i.e.*, 4th and 5th terms of AP are

$a_4 = a_1 + (4 - 1)\, d = 5 + 3 \times 3 = 5 + 9 = 14$

and $a_5 = a_1 + (5 - 1)\, d = 5 + 4 \times 3 = 5 + 12 = 17$

6. (d) Given, $d = -4$, $n = 7$ and $a_n = 4$

In an AP, nth term of AP (a_n)

$$= a + (n - 1)d$$

$\therefore \quad 4 = a + (7-1)(-4)$

$\Rightarrow \quad 4 = a + (6)(-4)$

$\Rightarrow \quad 4 = a - 24$

$\therefore \quad a = 4 + 24 = 28$

7. (b) Let the first term and common difference of an AP be 'a' and 'd' respectively.

TiP

In an AP, $a_n = a + (n - 1)d$

Given, $a = 28$ and $d = -4$

$\therefore$ 7th term, $a_7 = a + (7 - 1)d = 28 + 6 \times (-4)$

$$= 28 - 24 = 4$$

8. (c) Let the first term and common difference of an AP be 'a' and 'd' respectively.

Suppose n be the number of terms of given AP.

Given, $\quad a = 14$, $d = 19 - 14 = 5$

and $\quad a_n = 119$

$\because \quad a_n = a + (n - 1)\, d$

$\therefore \quad 119 = 14 + (n - 1)(5)$

$\Rightarrow \quad 105 = 5n - 5$

$\Rightarrow \quad 5n = 110$

$\therefore \quad n = \frac{110}{5} = 22$

9. (b) Given, first term $(A) = a$,

common difference $(D) = 3a - a = 2a$

$\because$ nth term of AP,

$$a_n = A + (n - 1)D$$

$\therefore \; a_n = a + (n - 1)2a = a + 2an - 2a$

$$= 2an - a = (2n - 1)a$$

10. (b) Given AP sequence is 21, 42, 63, 84, ...

Here, $\quad$ first term, $a = 21$

Common difference, $d = 42 - 21 = 21$

and last term, $l = 210$

Then, nth term of an AP sequence is given by

$$T_n = l = a + (n - 1)d$$

$\therefore \quad 210 = 21 + (n - 1)21$

$\Rightarrow \quad 210 = 21 + 21n - 21$

$\Rightarrow \quad 21\, n = 210$

$\Rightarrow \quad n = 10$

Hence, 10th term of an AP is 210.

11. (b) Given, $a_1 = -3$ and $a_2 = 4$

Here, common difference, $d = a_2 - a_1 = 4 - (-3) = 7$

and first term, $a = -3$

Then nth term of an AP is given by

$$T_n = a + (n - 1)d$$

Therefore, 21st term of an AP is

$$T_{21} = -3 + (21 - 1)(7)$$

$$= -3 + 20 \times 7 = -3 + 140 = 137$$

12. (c) Let a be the first term and d be the common difference of the given AP.

Given, $d = 5$

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nth term of an AP is given by

$$a_n = a + (n - 1)d$$

$\therefore \quad a_{18} - a_{13} = a + (18 - 1)d - [a + (13 - 1)d]$

$$= 17d - 12d = 5d = 5 \times 5 = 25$$

13. (b) Given AP is: 17, 14, 11, ... , -40

Here, first term, $a = 17$.

Common difference, $d = 14 - 17 = -3$

and last term, $l = -40$

TR!CK

nth term from the end of an AP is $l - (n - 1)d$.

$\therefore$ 7th term from the end of an AP

$$= -40 - (7 - 1)(-3)$$

$$= -40 - 6(-3) = -40 + 18 = -22$$

14. (b) Let a and d be the first term and common difference of an AP respectively.

Then, nth term of an AP is

$$T_n = a + (n - 1)d$$

It is given that

$$T_2 = 13$$

and $T_5 = 25$

$\therefore \quad a + (2-1)d = 13$

$\Rightarrow \quad a + d = 13 \quad ...(1)$

and $\quad a + (5-1)d = 25$

$\Rightarrow \quad a + 4d = 25 \quad ...(2)$

Subtract eq. (1) from eq. (2), we get

$$(a + 4d) - (a + d) = 25 - 13$$

$\Rightarrow \quad 3d = 12 \quad \Rightarrow d = 4$

Put $d = 4$ in eq. (1), we get

$$a + 4 = 13 \quad \Rightarrow \quad a = 9$$

$\therefore$ 7th term of an AP is

$$T_7 = 9 + (7-1)4 = 9 + 6 \times 4 = 9 + 24 = 33$$

15. (d) Given, sum of first 'n' terms of an AP,

$$S_n = 3n^2 + n$$

Then, nth term of an AP is determined by

$$a_n = S_n - S_{n-1} = (3n^2 + n) - 3(n-1)^2 - (n-1)$$
$$= 3n^2 + n - 3(n^2 + 1 - 2n) - (n-1)$$
$$= 3n^2 + n - 3n^2 - 3 + 6n - n + 1$$
$$= 6n - 2$$

$\therefore$ First term of an AP is $a_1 = 6 \times 1 - 2$ (put $n = 1$)

$$= 6 - 2 = 4$$

16. (b) Let the first term and common difference of an AP be 'a' and 'd' respectively.

TiP

Sum of first n terms of an AP

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Given, $\quad a = 16$ and $d = 12 - 16 = -4$

$$\therefore \quad S_{21} = \frac{21}{2}\{2a + (21-1)d\} = \frac{21}{2}\{2 \times 16 + 20(-4)\}$$
$$= \frac{21}{2}(32 - 80) = \frac{21}{2} \times (-48)$$
$$= 21 \times (-24) = -504$$

17. (a) Given, $a = 1$, $a_n = 20$ and $S_n = 399$.

The sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad 399 = \frac{n}{2}[2 \times 1 + (n-1)d]$$

$\Rightarrow \quad 798 = 2n + n(n-1)d \quad ...(1)$

Also, $\quad a_n = 20$

$\therefore \quad a + (n-1)d = 20$

$\Rightarrow \quad 1 + (n-1)d = 20$

$\Rightarrow \quad (n-1)d = 19$

Put $(n-1)d = 19$ in eq. (1), we get

$$798 = 2n + 19n$$

$\Rightarrow \quad 798 = 21n \quad \Rightarrow \quad n = 38$

18. (b) Given AP is 64, 60, 56, ...

Here, $a = 64$, $d = 60 - 64 = -4$

Let n be the number of terms in the given AP.

Then, $\quad S_n = 544$

$$\therefore \quad \frac{n}{2}[2a + (n-1)d] = 544$$

$$\Rightarrow \frac{n}{2}[2 \times 64 + (n-1)(-4)] = 544$$

$$\Rightarrow \quad \frac{n}{2} \times 2[64 - 2(n-1)] = 544$$

$\Rightarrow \quad n[66 - 2n] = 544$

$\Rightarrow \quad 2n^2 - 66n + 544 = 0$

$\Rightarrow \quad n^2 - 33n + 272 = 0$

$\Rightarrow \quad n^2 - (17 + 16)n + 272 = 0$

(by splitting the middle term)

$\Rightarrow \quad n^2 - 17n - 16n + 272 = 0$

$\Rightarrow \quad n(n-17) - 16(n-17) = 0$

$\Rightarrow \quad (n-16)(n-17) = 0$

$\Rightarrow \quad n - 16 = 0$ or $n - 17 = 0$

$\Rightarrow \quad n = 16$ or $n = 17$

19. (c) **Assertion (A):** Given sequence: $-5, -\frac{5}{2}, 0, \frac{5}{2}, ...$

Here, $a_1 = -5, a_2 = -\frac{5}{2}, a_3 = 0, a_4 = \frac{5}{2}$

Difference of two consecutive terms

$$a_2 - a_1 = \frac{-5}{2} - (-5) = -\frac{5}{2} + 5 = \frac{5}{2}$$

$$a_3 - a_2 = 0 - \left(-\frac{5}{2}\right) = \frac{5}{2}$$

$$a_4 - a_3 = \frac{5}{2} - 0 = \frac{5}{2}$$

Since, the difference of two consecutive terms is constant *i.e.*, $\frac{5}{2}$.

Therefore, given sequence is an AP.

So, Assertion (A) is true.

Reason (R): The terms of an AP. can have both positive and negative rational numbers.

So, Reason (R) is false.

Hence, Assertion (A) is true but Reason (R) is false.

20. (a) **Assertion (A):** Given sequence is $-8, -4, 0, 4,$ ....

$\because \quad a_2 - a_1 = -4 - (-8) = 4,$

$a_3 - a_2 = 0 - (-4) = 4,$

$a_4 - a_3 = 4 - 0 = 4$

Here, we see that difference of two consecutive terms is same constant. So, given sequence is an AP.

$\therefore \quad$ First term, $a = -8$

and common difference, $d = 4$

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nth term of an AP is

$$T_n = a + (n-1)d$$

$$\therefore \quad T_n = -8 + (n-1)(4)$$
$$= -8 + 4n - 4 = 4n - 12$$

So, Assertion (A) is true.

Reason (R): It is also true that nth term of an AP is determined by $T_n = a + (n-1)d$.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

21. (b) **Assertion (A):** Let a and d be the first term and common difference of an AP. Then, nth term of an AP is

$$a_n = a + (n-1)d$$

Given, $a_{20} - a_{16} = 20$

$\therefore [a + (20-1)d] - [a + (16-1)d] = 20$

$$19d - 15d = 20$$

$\Rightarrow \quad 4d = 20$

$\Rightarrow \quad d = 5$

So, Assertion (A) is true.

Reason (R): Given sequence is

$$\sqrt{2}, \sqrt{4}, \sqrt{18} \_\_\_$$

or $\quad \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}$ ....

Here $a = \sqrt{2}, d = 2\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$\because \quad T_n = a + (n-1)d$

$\therefore \quad T_n = \sqrt{2} + (n-1)\sqrt{2} = \sqrt{2}n$

So, Reason (R) is also true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

22. (b) **Assertion (A):**

If part: Given a, b, c are in AP.

Then $\quad b - a = c - b$

$\Rightarrow \quad b + b = a + c \Rightarrow 2b = a + c$

Only part: Given, $2b = a + c$

$\Rightarrow \quad b + b = a + c$

$\Rightarrow \quad b - a = c - b$

$\Rightarrow \quad a_2 - a_1 = a_3 - a_2 \quad$ (let $a_1 = a, a_2 = b$ and $a_3 = c$)

Since, each term differs from its preceding term are equal.

$\therefore$ The sequence a_1, a_2, a_3 or a, b, c are in AP.

Therefore, a, b, c are in AP if and only if $2b = a + c$.

So, Assertion (A) is true.

Reason (R): First n odd natural numbers are:

1, 3, 5, 7,...

Here, first term $(a) = 1$

and common difference $(d) = 3 - 1 = 5 - 3 = 2$

Since, the difference between each consecutive terms is constant *i.e.*, 2.

So, the sequence forms an AP.

$\therefore$ Sum of first n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2 \times 1 + (n-1) \times 2]$$

$$= \frac{n}{2} \times 2(1 + n - 1) = n \cdot n = n^2$$

So, Reason (R) is true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

23. (c) **Assertion (A):** Given, $S_n = 6n^2 - 2n$.

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nth term of an AP, whose sum is S_n, is

$$T_n = S_n - S_{n-1}$$

Using formula,

$$T_n = S_n - S_{n-1} = (6n^2 - 2n) - [6(n-1)^2 - 2(n-1)]$$
$$= 6n^2 - 2n - [6(n^2 + 1 - 2n) - 2n + 2]$$
$$= 6n^2 - 2n - [6n^2 - 14n + 8]$$
$$= -2n + 14n - 8 = 12n - 8$$

So, Assertion (A) is true.

Reason (R): It is not true that

$$T_n = S_{n-1} - S_n$$

Thus, the correct relation is

$$T_n = S_n - S_{n-1}$$

Hence, Assertion (A) is true but Reason (R) is false.

24. Zero.

25. Given, three consecutive terms of an AP are $k^2 + 4k + 8$, $2k^2 + 3k + 6$ and $3k^2 + 4k + 4$.

Therefore, common difference of each term of an AP is equal.

Here, $\quad a_1 = k^2 + 4k + 8, a_2 = 2k^2 + 3k + 6$

and $\quad a_3 = 3k^2 + 4k + 4$

$\therefore \quad a_2 - a_1 = a_3 - a_2$

$\Rightarrow (2k^2 + 3k + 6) - (k^2 + 4k + 8)$

$$= (3k^2 + 4k + 4) - (2k^2 + 3k + 6)$$

$\Rightarrow \quad k^2 - k - 2 = k^2 + k - 2$

$\Rightarrow \quad 2k = 0 \Rightarrow k = 0$

26. Let a be the first term and d be the common difference of an AP. Then, according to the given condition,

$$7\,T_7 = 11\,T_{11}$$

$\therefore 7\,[a + (7-1)d] = 11\,[a + (11-1)d] \quad [\because T_n = a + (n-1)d]$

$\Rightarrow \quad 7[a + 6d] = 11\,[a + 10d]$

$\Rightarrow \quad 7a + 42d = 11a + 110d$

$\Rightarrow \quad 4a + 68d = 0$

$\Rightarrow \quad a + 17d = 0 \quad$ ...(1)

Now, the 18th term of an AP is

$$T_{18} = a + (18-1)d$$
$$= a + 17d = 0 \quad \text{[from eq. (1)]}$$

27. Let nth term of the AP : 24, 21, 18, 15, ... be the first negative term.

Here, first term $(a) = 24$

and common difference $(d) = 21 - 24 = -3$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad [a + (n-1)d] < 0$

$\Rightarrow \quad [24 + (n-1)(-3)] < 0$

$\Rightarrow \quad (24 - 3n + 3) < 0$

$\Rightarrow \quad 27 - 3n < 0$

$\Rightarrow \quad 3n > 27 \Rightarrow n > 9$

$\therefore \quad n = 10$

So, 10th term is the first negative term.

COMMON ERR!R

Sometimes students take the value of $n = 9$ instead of taken the value of $n = 10$.

28. Given AP is $-8, -6, -4,$ ....

Here, first term, $a = -8$

and common difference, $d = -6 - (-8) = 2$

Then, sum of an AP is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2 \times (-8) + (10-1)(2)]$$

$$= 5[-16 + 18] = 5 \times 2 = 10$$

29. True

30. Given, $a = 1$, $a_n = 20$ and $n = 38$

$\because$ $$S_n = \frac{n}{2}(a + a_n)$$

$$S_{38} = \frac{38}{2}(1 + 20)$$

$\Rightarrow$ $S_{38} = 19 \times 21$

$\Rightarrow$ $S_{38} = 399$

Hence, given statement is false.

31. Let a and d be the first term and common difference respectively. Then

$a = -5$ and $d = 2$

The sum of first n terms of an AP is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_6 = \frac{6}{2}[2 \times (-5) + (6-1)2]$$

$$= 3[-10 + 10] = 0$$

Hence, given statement is false.

32. Given, AP is 18, 16, 14, ...

Here, $a = 18$, $d = 16 - 18 = -2$

Now, sum of 20 terms of an AP is

$$S_{20} = \frac{20}{2}[2 \times 18 + (20-1)(-2)]$$

$$\left[\because S_n = \frac{n}{2}(2a + (n-1)d)\right]$$

$$= 10[36 - 38] = 10 \times (-2) = -20$$

Hence, given statement is false.

33. Given, $S_n = 3n^2 + 4$

$\therefore$ nth term, $a_n = S_n - S_{(n-1)}$

$= 3n^2 + 4 - [3(n-1)^2 + 4]$

$= 3n^2 + 4 - [3(n^2 + 1 - 2n) + 4]$

$= 3n^2 + 4 - 3n^2 + 6n - 7$

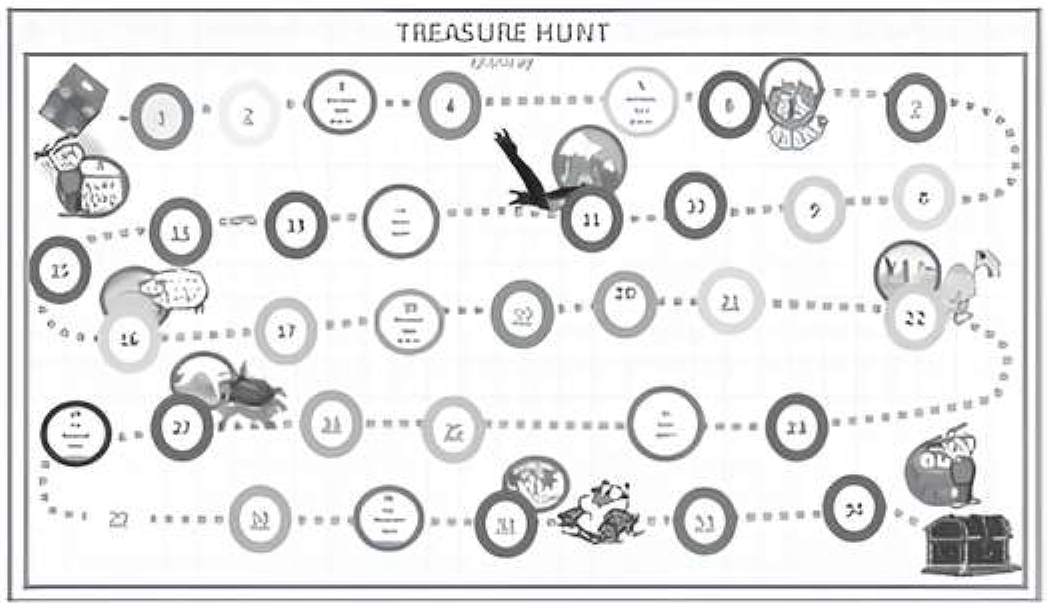
$= 6n - 3$

Hence, given statement is true.

Case Study Based Questions

Case Study 1

While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forms an AP. If the number on the nth spot is $20 + 4n$, then answer the following questions to help the player in spotting the clues.



Based on the given information, solve the following questions:

Q 1. Which number is on the first spot?

a. 20　　b. 24

c. 16　　d. 28

Q 2. The common difference of the given AP, is:

a. 20　　b. 4

c. 16　　d. 24

Q 3. Which number is on the $(n-2)$th spot?

a. $16 + 4n$　　b. $24 + 4n$

c. $12 + 4n$　　d. $28 + 4n$

Q 4. What is the sum of all the numbers on the first 10 spots?

a. 410　　b. 420

c. 480　　d. 430

Q 5. Which spot is numbered as 116?

a. 5th　　b. 8th

c. 9th　　d. 24th

Solutions

1. Given, number on nth spot $= 20 + 4n$

i.e., $a_n = 20 + 4n$

$\therefore$ Number on first spot, $a_1 = 20 + 4(1)$

$= 20 + 4 = 24$

So, option (b) is correct.

2. The common difference $(d) = a_n - a_{n-1}$

$\Rightarrow d = 20 + 4n - \{20 + 4(n-1)\}$　　$[\because a_n = 20 + 4n]$

$= 20 + 4n - 20 - 4n + 4 = 4$

So, option (b) is correct.

3. The number on $(n-2)$th spot,

$a_{n-2} = 20 + 4(n-2)$　　$[\because a_n = 20 + 4n]$

$= 20 + 4n - 8$

$= 12 + 4n$

So, option (c) is correct.

4. $\because$ The sum of first n term of an AP

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$\therefore$ The sum of all the numbers on the first 10 spots,

$$S_{10} = \frac{10}{2}\{2 \times 24 + (10-1) \times 4\}$$

$= 5(48 + 36)$

$= 5 \times 84 = 420$

So, option (b) is correct.

5. (d) Let nth spot is numbered as 116.

$\therefore \quad a_n = 116 \quad \Rightarrow \quad 20 + 4n = 116$

$\Rightarrow \quad 4n = 96 \quad \Rightarrow \quad n = 24.$

$\therefore$ 24th spot is numbered as 116.

So, option (d) is correct.

Case Study 2

Your younger sister wants to buy an electric car and plans to take loan from a bank for her electric car. She repays her total loan of ₹ 321600 by paying every month starting with the first instalment of ₹ 2000 and it increases the instalment by ₹ 200 every month.

Based on the above information, solve the following questions:

Q 1. Find the list of the instalment formed by the given statement.

a. 2000, 1800, 1600,... b. 2000, 2200, 2400,...

c. 2200, 2400, 2600,... d. 2300, 2600, 2900,...

Q 2. The amount paid by her in 25th instalment is:

a. ₹ 6800 b. ₹ 3500 c. ₹ 4800 d. ₹ 6600

Q 3. Find the difference of the amount in 4th and 6th instalment paid by younger sister.

a. ₹ 200 b. ₹ 400 c. ₹ 600 d. ₹ 800

Q 4. In how many instalment, she clear her total bank loan?

a. 1582 b. 1580 c. 1599 d. 1600

Q 5. Find the sum of the first seven instalments.

a. ₹ 14000 b. ₹ 13600 c. ₹ 10400 d. ₹ 12600

Solutions

1. It can be observed that these instalments are in AP having first term (instalment) as ₹ 2000 and common difference (increase instalment) as ₹200.

Here, $a = 2000$ and $d = 200$

Therefore list of an AP is $a, a + d, a + 2d, \dots$

i.e., $2000, 2000 + 200, 2000 + 2 \times 200, \dots$

i.e., $2000, 2200, 2400, \dots$

So, option (b) is correct.

2. It can be observed that these instalments are in an AP having first term (instalment) as ₹ 2000 and common difference (increase instalment) as ₹ 200.

Here, $a = 2000$ and $d = 200$

TR!CK

nth term of an AP is, $T_n = a + (n - 1)\, d$ where, a and d are first term and common difference respectively.

$\therefore$ The amount paid by her in 25th instalment is

$$T_{25} = a + (25 - 1)\, d$$
$$= 2000 + 24 \times 200$$
$$= 2000 + 4800 = ₹\, 6800$$

So, option (a) is correct.

3. Let a and d be the first term and common difference of an AP.

Then, $a_4 = a + (4 - 1)d \quad [\because a_n = a + (n - 1)d]$

$= a + 3d$

Similarly, $a_6 = a + 5d$.

$\therefore$ Required difference $= a_6 - a_4$

$= (a + 5d) - (a + 3d) = 2d$

$= 2 \times 200 = ₹\, 400$

So, option (b) is correct.

4. Let in n instalments, she clear her loan.

Given, $T_n = 321600$

$\because \quad T_n = a + (n - 1)d$

$\therefore \quad 321600 = 2000 + (n - 1)200$

$\Rightarrow \quad 319600 = (n - 1)200$

$\Rightarrow \quad 1598 = n - 1$

$\Rightarrow \quad n = 1599$

So, in 1599 instalments, she clear her bank loan.

So, option (c) is correct.

5. Here, $a = 1200$, $d = 200$

$\therefore$ The sum of first seven instalments is

$$S_7 = \frac{7}{2}[2 \times 1200 + (7 - 1)200]$$

$$\left[\because S_n = \frac{n}{2}[2a + (n - 1)d]\right]$$

$$= \frac{7}{2}[2400 + 1200]$$

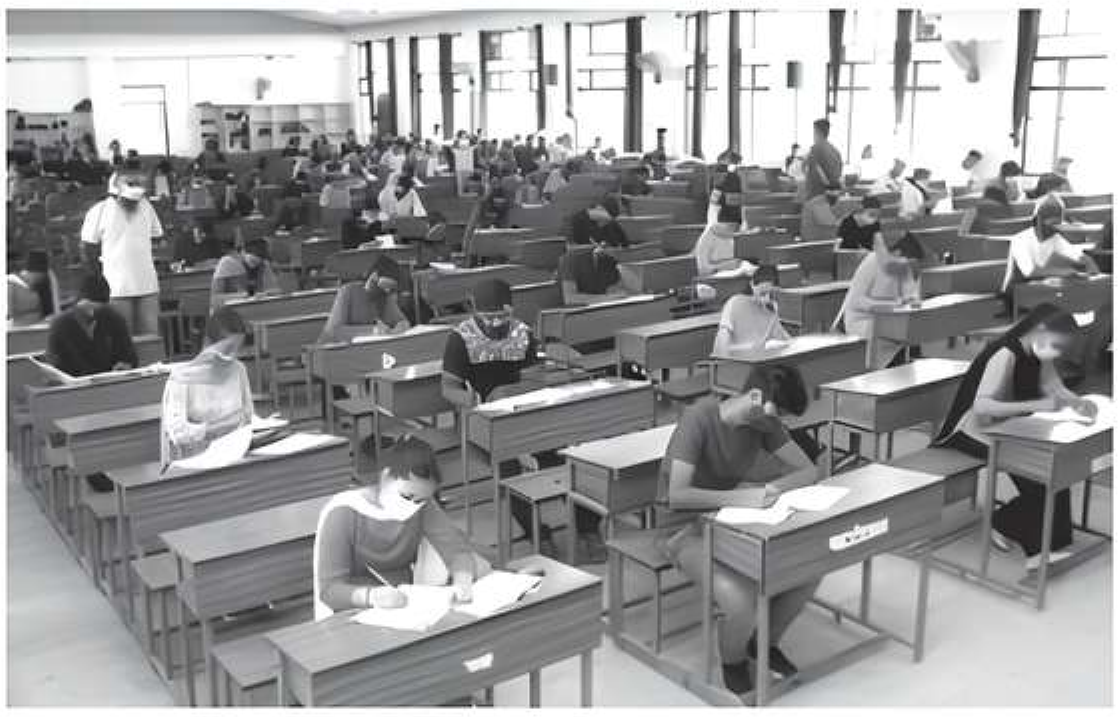
$$= \frac{7}{2}[3600] = 7 \times 1800 = ₹\, 12600$$

So, option (d) is correct.

Case Study 3

In an examination hall, the examiner makes students sit in such a way that no students can cheat from other student and make no student sit uncomfortably. So, the teacher decides to mark the numbers on each chair from 1, 2, 3, ...

There are 25 students and each student is seated at alternate position in examination room such that the sequence formed is 1, 3, 5, ...



Based on the given information, solve the following questions:

Q 1. What type of sequence is formed, to follow the seating arrangement of students in the examination room?

Q 2. Find the seat number of the last student in the examination room.

Q 3. Find the seat number of 10th vacant seat in the examination room.

OR

Which number of student will seat on the 27th seat number.

Solutions

1. Given, seating arrangement of students in the examination room is 1, 3, 5, ...

Here, $a_1 = 1, a_2 = 3, a_3 = 5, \ldots\ldots$

Now, $a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 5 - 3 = 2$

Here, common difference is same, so given sequence is the type of Arithmetic progression.

2. Given, $a = 1, d = 3 - 1 = 2$ and $n = 25$

$\because \quad T_n = a + (n-1)d$

There are 25 students.

$\therefore \quad T_{25} = 1 + (25-1)2 = 1 + 24 \times 2 = 49$

Hence, last student will sit on the 49th seat number.

3. The sequence of vacant seats are as follows, 2, 4, 6, ..., 48.

Here, $a = 2, d = 4 - 2 = 6 - 4 = 2$

The 10th vacant seat will be

$T_{10} = a + (10-1)d \qquad [\because T_n = a + (n-1)d]$

$= 2 + 9 \times 2 = 2 + 18 = 20$

Hence, the 10th vacant seat number is 20.

OR

The sequence of seating arrangement of students in the examination room is

1, 3, 5, 7....

Let nth number of student will seat on 27th seat number.

Here, $a = 1, d = 3 - 1 = 2$ and $T_n = 27$

$\because \quad T_n = a + (n-1)d$

$\therefore \quad 27 = 1 + (n-1)(2)$

$\Rightarrow \quad n - 1 = \frac{26}{2}$

$\Rightarrow \quad n = 13 + 1 = 14$

So, 14th student will sit on 27th seat number.

Case Study 4

In a pathology lab, a culture test has been conducted. In the test, the number of bacteria taken into consideration in various samples is all 3-digit numbers that are divisible by 7, taken in order.

Based on the above information, solve the following questions:

Q 1. How many bacteria are considered in the seventh sample?

Q 2. How many samples should be taken into consideration?

Q 3. Find the total number of bacteria in the first 15 samples.

OR

Find the number of samples in which sum of bacteria is 840.

Solutions

1. Here the smallest 3-digit number divisible by 6 is 102 and the largest 3-digit number divisible by 6 is 996. So, the number of bacteria taken into consideration is 102, 108, 114, ... , 996.

Here, first term $(a) = 102$, common difference (d)
$= 108 - 102 = 6$

and last term $(a_n) = 996$.

$\therefore$ Seventh sample, $a_7 = a + (7-1)d$

$[\because a_n = a + (n-1)d]$

$= 102 + 6 \times 6$

$= 102 + 36 = 138$

2. Let n samples be taken under consideration.

Here, last term $(a_n) = 996$.

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad 996 = 102 + (n-1)6$

$\Rightarrow \quad n - 1 = \frac{894}{6} = 149 \Rightarrow n = 150$

3. We know that,

Sum of first n terms of the AP $S_n = \frac{n}{2}\,[2a + (n-1)d]$

$\therefore$ Total number of bacteria in first 15 samples.

$$S_{15} = \frac{15}{2}\{2 \times 102 + (15-1) \times 6\}$$

$$= 15\,(102 + 14 \times 3) = 15 \times 144 = 2160$$

OR

Let n number of samples should be taken in which sum of bacteria is 840.

Here, $a = 102$, $d = 108 - 102 = 6$ and $S_n = 840$.

$$\because \quad S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$840 = \frac{n}{2}\{2 \times 102 + (n-1)6\}$$

$$\Rightarrow \quad 840 = n\,(102 + 3n - 3)$$

$$\Rightarrow \quad 3n^2 + 99n - 840 = 0$$

$$\Rightarrow \quad n^2 + 33n - 280 = 0$$

$$\Rightarrow \quad n^2 + 40n - 7n - 280 = 0$$

$$\Rightarrow \quad n\,(n+40) - 7\,(n+40) = 0$$

$$\Rightarrow \quad (n+40)\,(n-7) = 0$$

$$n = -40, 7$$

But samples cannot be negative.

$$\therefore \quad n = 7$$

So, required number of sample is 7.

Case Study 5

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

Based on the above information, solve the following questions: *[CBSE SQP 2023-24]*

Q 1. In which year, the production is Rs 29,200.

Q 2. Find the difference of the production during 7th year and 4th year.

Q 3. Find the production during 8th year.

OR

Find the production during first 3 years.

Solutions

Given that, the production of TV sets in a factory increases uniformly by a fixed number every year i.e., production of TV sets in every year form an AP.

Let the first term and common difference of this AP be 'a' and 'd' respectively.

According to the question, the factory produced 16000 sets in 6th year and 22600 in 9th year.

TR!CK

nth term of an AP is $T_n = a_n = a + (n-1)d$

i.e., $\quad a_6 = a + (6-1)d = 16000$

$\Rightarrow \quad a + 5d = 16000 \quad$ ...(1)

and $\quad a_9 = a + (9-1)d = 22600$

$\Rightarrow \quad a + 8d = 22600 \quad$ ...(2)

Subtract eq (1) from eq (2), we get

$$(a + 8d) - (a + 5d) = 22600 - 16000$$

$$\Rightarrow \quad 3d = 6600$$

$$\Rightarrow \quad d = \frac{6600}{3} = 2200$$

put the value of 'd' in eq. (1), we get

$$a + 5\,(2200) = 16000$$

$$\Rightarrow \quad a = 16000 - 11000$$

$$\Rightarrow \quad a = 5000.$$

1. Let nth year, the production is,

$$a_n = 29{,}200$$

$$\Rightarrow \quad a + (n-1)d = 29200$$

$$\Rightarrow \quad 5000 + (n-1)\,(2200) = 29200$$

$$\Rightarrow \quad (n-1) = \frac{24200}{2200} = 11$$

$$\therefore \quad n = 11 + 1 = 12$$

Hence, in 12th year, the production is ₹ 29,200.

2. Now, the production during 7th year is

$$a_7 = a + (7-1)d$$
$$= 5000 + 6(2200)$$
$$= 5000 + 13200 = 18200$$

and the production during 4th year is

$$a_4 = a + (4-1)d$$
$$= 5000 + 3(2200)$$
$$= 5000 + 6600 = 11600.$$

$\therefore$ The difference of the production during 7th year and 4th year $= 18200 - 11600 = 6600$.

3. The production during 8th year is

$$a_8 = a + (8-1)d$$
$$= 5000 + 7 \times 2200$$
$$= 5000 + 15400$$
$$= 20400.$$

OR

TR!CK

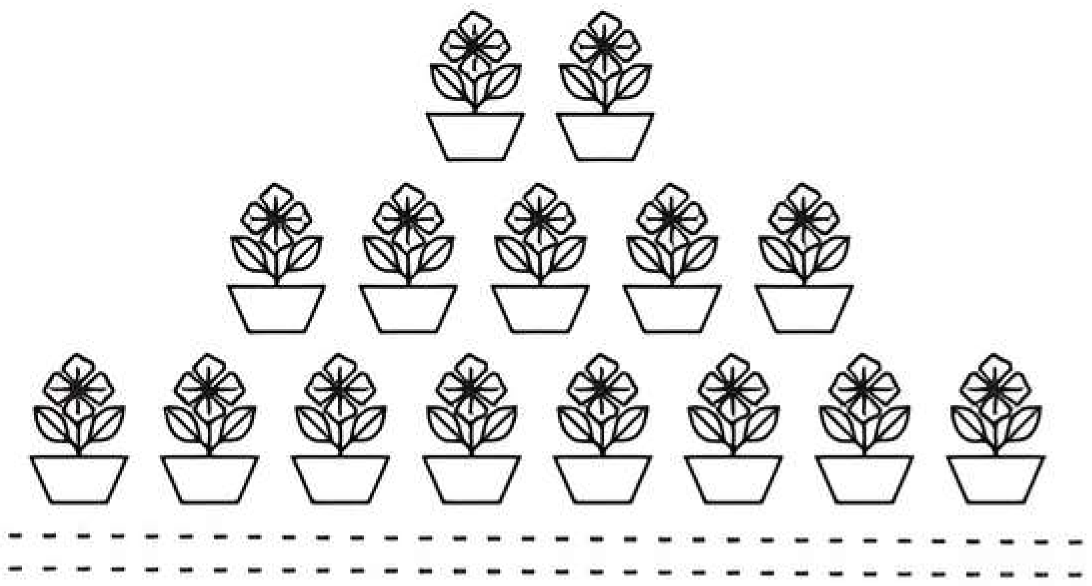
The sum of n terms of an AP is $S_n = \frac{n}{2}[2a + (n-1)d]$.

The production during first 3 years is

$$S_3 = \frac{3}{2}[2 \times 5000 + (3-1)(2200)]$$
$$= 3\,(5000 + 2200)$$
$$= 3 \times 7200 = 21600.$$

Case Study 6

Aahana being a plant lover decides to convert her balcony into beautiful garden full of plants. She bought few plants with pots for her balcony. She placed the pots in such a way that number of pots in the first row is 2, second row is 5, third row is 8 and so on.



Based on the above information, solve the following questions: [CBSE 2023]

Q 1. Find the number of pots placed in the 10th row.

Q 2. Find the difference in the number of pots placed in 5th row and 2nd row.

Q 3. If Aahana wants to place 100 pots in total, then find the total number of rows formed in the arrangement.

OR

If Aahana has sufficient space for 12 rows, then how many total number of pots are placed by her with the same arrangement?

Solutions

Given that, Aahana placed the pots in such a way that number of pots in the first row is 2, second row is 5, third row is 8 and so on. Such a pattern form a sequence:

2, 5, 8,...

Here, $5-2 = 8-5 = 3$ (Constant)

So, the above sequence form an AP.

Let the first term and common difference of this AP. be 'a' and 'd' respectively.

TR!CK

nth term of an AP is $T_n = a_n = a + (n-1)d$

Here, $a = 2$ and $d = 3$

1. The number of pots placed in the 10th row is,

$$a_{10} = a + (10-1)d$$
$$= 2 + 9(3)$$
$$= 2 + 27 = 29.$$

2. The number of pots placed in 5th row is,

$$a_5 = a + (5-1)d$$
$$= 2 + 4 \times 3 = 2 + 12 = 14.$$

and the number of pots placed in 2nd row is,

$$a_2 = a + (2-1)d$$
$$= a + d = 2 + 3 = 5.$$

$\therefore$ The difference in the number of pots placed in 5th row and 2nd row

$$= 14 - 5 = 9.$$

3. Let 'n' number of rows formed in the arrangement

Here, $S_n = 100$, $a = 2$ and $d = 3$

TR!CK

The sum of n terms of an AP is $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore \quad 100 = \frac{n}{2}\{2 \times 2 + (n-1)(3)\}$$
$$\Rightarrow \quad 200 = n\,(4 + 3n - 3)$$
$$\Rightarrow \quad 3n^2 + n - 200 = 0$$
$$\Rightarrow \quad 3n^2 + 25n - 24n - 200 = 0$$

(By splitting the middle terms)

$$\Rightarrow \quad n(3n+25) - 8(3n+25) = 0$$
$$\Rightarrow \quad (3n+25)\,(n-8) = 0$$
$$\Rightarrow \quad 3n + 25 = 0 \quad \text{or} \quad n - 8 = 0$$
$$\Rightarrow \quad n = \frac{-25}{3}, 8.$$

But n cannot be negative because n is a natural number.

$$\therefore \quad n = 8$$

So, required number of rows is 8.

OR

Given, $n = 12$, $a = 2$ and $d = 3$

$$\because \quad S_n = \frac{n}{2}\{2a + (n-1)d\}$$
$$S_{12} = \frac{12}{2}\{2 \times 2 + (12-1)3\}$$
$$= 6\,(4 + 33) = 222$$

So, the total number of pots are 222, placed by her with the same arrangement.

Very Short Answer Type Questions

Q 1. Write the common difference of the AP:

$$\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots.$$

[NCERT EXEMPLAR; CBSE 2019]

Q 2. Suppose $a = 3$, $d = -2$, $l = -11$, find the number of terms exist in an AP.

Q 3. What is the common difference of an AP in which $a_{21} - a_7 = 84$? [CBSE 2017]

Q 4. In an AP, if the common difference (d) is -4 and the seventh term (a_7) is 4, then find the first term. [CBSE 2018]

Q 5. How many two digit numbers are divisible by 3? [NCERT EXERCISE; CBSE 2019]

Q 6. Find the 9th term from the end (towards the first term) of the AP: 5, 9, 13, ..., 185. [CBSE 2016]

Q 7. If $a = 2$ and $d = 3$, then find the sum of first 10 terms of an AP.

Q 8. How many terms of AP: 18, 16, 14, ... should be taken so that their sum is zero?

Q 9. If nth term of an AP is $(2n + 3)$, what is the sum of its first five terms?

Short Answer Type-I Questions

Q 1. Find whether −150 is a term of the AP: 17, 12, 7, 2, ... [V.Imp.]

Q 2. Which term of the AP: 8, 14, 20, 26, ... will be 72 more than its 41st term? [CBSE 2017]

Q 3. In an AP, if the sum of third and seventh terms is zero, find its 5th term. [CBSE 2022 Term-II]

Q 4. If 7 times the seventh term of the AP is equal to 5 times the fifth term, then find the value of its 12th term. [CBSE 2022 Term-II]

Q 5. Determine the AP whose third term is 5 and seventh term is 9. [CBSE 2022 Term-II]

Q 6. Determine the AP whose third term is 16 and seventh term exceeds the fifth term by 12. [NCERT EXERCISE; CBSE 2019]

Q 7. Find the sum of first 30 terms of AP: −30, −24, −18, ... [CBSE 2022 Term-II]

Q 8. In an AP, if $S_n = n(4n+1)$, then find the AP. [CBSE 2022 Term-II]

Q 9. Find the sum of first 20 terms of an AP, whose nth term is given as $a_n = 5 - 2n$.

Q 10. If S_n, the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the nth term and common difference. [CBSE 2019]

Short Answer Type-II Questions

Q 1. How many terms are there in AP whose first and fifth terms are −14 and 2, respectively and the last term is 62. [CBSE 2023]

Q 2. Which term of the AP : 65, 61, 57, 53, ... is the first negative term? [CBSE 2023]

Q 3. The 24th term of an AP is twice its 10th term. Show that its 72nd term is 4 times its 15th term.

Q 4. If the 3rd and the 9th terms of an AP are 4 and −8 respectively, which term of this AP is zero? [NCERT EXERCISE; V. Imp.]

Q 5. The sum of the 5th and the 9th terms of an AP is 30. If its 25th term is three times its 8th term, find the AP. [V. Imp.]

Q 6. Each year, a tree grows 5 cm less than it grew the preceding year. If it grew by 1 m in the first year, then in how many years will it have ceased growing? [CBSE 2015]

Q 7. Find the number of terms of the AP: 18, $15\frac{1}{2}$, 13, ..., $-49\frac{1}{2}$ and find the sum of all its terms.

Q 8. The sum of first 15 terms of an AP is 750 and its first term is 15. Find its 20th term. [CBSE 2023]

Q 9. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference. [NCERT EXERCISE; CBSE 2017]

Q 10. How many terms of the AP: 54, 51, 48, ... should be taken so that their sum is 513? Explain the double answer.

Q 11. Show that the sum of all terms of an AP whose first term is a, the second term is b and the last term is c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$. [NCERT EXEMPLAR; CBSE 2020]

Long Answer Type Questions

Q 1. A man repays a loan of ₹ 3,250 by paying ₹ 20 in the first month and then increases the payment by ₹ 15 every month. How long will it take to clear the loan? [CBSE 2023]

Q 2. In an AP, the sum of the first 'n' terms is $3n^2 + n$. Find the first term and the common difference of the AP. Hence, find its 15th term. [CBSE 2023]

Q 3. The sum of the first 8 terms of an AP is 100 and the sum of its first 19 terms is 551. Find the sum of its first 'n' terms. [CBSE 2023]

Q 4. Find the number of terms of the AP: −12, −9, −6, ..., 21. If 1 is added to each term of this AP, then find the sum of all terms of the AP thus obtained.

Q 5. If the sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms. [CBSE 2023]

Q 6. If the sum of the first p terms of an AP is the same as the sum of its first q terms (where $p \neq q$), then show that the sum of first $(p + q)$ terms is zero. [CBSE 2019, 23]

Solutions

Very Short Answer Type Questions

1. Given, AP sequence is $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, ...$

or $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, ...$

Now, common difference,

d = Second term − First term

$= 2\sqrt{3} - \sqrt{3}$
$= \sqrt{3}$

2. Given, $a = 3$, $d = -2$ and $l = -11$

TR!CK

nth term of an AP is: $a_n = l = a + (n-1)d$.

$\therefore \quad -11 = 3 + (n-1)(-2)$
$\Rightarrow \quad -14 = (n-1)(-2)$
$\Rightarrow \quad 7 = n - 1$
$\Rightarrow \quad n = 8$

Hence, number of terms exist in an AP is 8.

3. Let the first term and common difference of an AP be a and d respectively.

Given, $\quad a_{21} - a_7 = 84$

$\Rightarrow [a + (21-1)d] - [a + (7-1)d] = 84$

[$\because$ nth term of AP, $a_n = a + (n-1)d$]

$\Rightarrow \quad a + 20d - a - 6d = 84$
$\Rightarrow \quad 14d = 84$
$\Rightarrow \quad d = \frac{84}{14} \Rightarrow d = 6$

4. Let 'a' be the first term and 'd' be the common difference of AP.

Given, seventh term $(a_7) = 4$

$\therefore \quad a + (7-1)d = 4$

[$\because$ nth term of AP, $a_n = a + (n-1)d$]

$\Rightarrow \quad a + 6(-4) = 4 \quad$ [$\because d = -4$, given]
$\Rightarrow \quad a - 24 = 4$
$\therefore \quad a = 28$

Hence, required first term is 28.

5. The two-digit numbers divisible by 3 are 12, 15, 18,....,99.

Here, common difference

$= 15 - 12 = 18 - 15 = 3$

So, it forms an AP.

Here, $\quad a = 12$, $d = 3$ and $l = 99$

$\because \quad a_n = l = a + (n-1)d$
$\therefore \quad 99 = 12 + (n-1)3$
$\Rightarrow \quad 3(n-1) = 87$
$\Rightarrow \quad n - 1 = 29 \Rightarrow n = 30$

Hence, the two-digit numbers divisible by 3 are 30.

6.

TR!CK

pth term from the end = (n − p + 1)th term from the beginning, where n is the number of terms.

Given AP is 5, 9, 13, ..., 185.

Here, $a = 5$, $d = 9 - 5 = 4$ and $a_n = 185$.

$\because \quad a_n = a + (n-1)d$
$\therefore \quad 185 = 5 + (n-1)(4)$
$\Rightarrow \quad 180 = 4(n-1) \Rightarrow n - 1 = 45$
$\Rightarrow \quad n = 46$

Now, 9th term from the end = (46 − 9 + 1)th term from the beginning

= 38th term from the beginning

$= a + 37d = 5 + 37 \times 4 \quad [\because a_n = a + (n-1)d]$
$= 5 + 148 = 153$

Hence, 9th term from the end is 153.

7. Given, $a = 2$, $d = 3$ and $n = 10$.

TR!CK

The sum of first n terms of an AP is $S_n = \frac{n}{2}[2a + (n-1)d]$.

$\therefore S_{10} = \frac{10}{2}[2 \times 2 + (10-1) \times 3] = 5[4 + 9 \times 3]$
$= 5[4 + 27] = 5 \times 31 = 155$

Hence, sum of first 10 terms of an AP is 155.

8. Let the sum of n terms of an AP be zero, i.e., $S_n = 0$.

Here, $\quad a = 18$ and $d = 16 - 18 = -2$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d] \quad ...(1)$

$\therefore \quad 0 = \frac{n}{2}[2 \times 18 + (n-1)(-2)] \quad$ [from eq. (1)]

$\Rightarrow n(36 - 2n + 2) = 0 \Rightarrow 38n - 2n^2 = 0$
$\Rightarrow \quad -2n(n-19) = 0 \Rightarrow n = 0$ or $n = 19$

But $n = 0$ (not possible)

Hence, the required number of terms is 19.

COMMON ERR(!)R

Some students consider both values of n in the answer but it is wrong approach. So, be careful about this.

9. Given, nth term of an AP is

$T_n = (2n + 3)$

Here, $a = T_1 = 2(1) + 3 = 5$
$l = T_5 = 2(5) + 3 = 13$

TR!CK

The sum of n terms of an AP is $S_n = \frac{n}{2}[a + l]$, where l is the last term of an AP.

$\because \quad S_n = \frac{n}{2}(a + l)$

$\therefore \quad S_5 = \frac{5}{2}[5 + 13] = \frac{5}{2} \times 18 = 45$

Hence, sum of first five terms of an AP is 45.

Short Answer Type-I Questions

1. Let nth term of the AP be −150.

Here, $\quad a = 17$, $d = 12 - 17 = -5$ and $a_n = -150$.

$\because \quad$ nth term of AP, $\quad a_n = a + (n-1)d$

$\therefore$ $-150 = 17 + (n-1)(-5)$

$\Rightarrow$ $-150 = 17 - 5n + 5$

$\Rightarrow$ $5n = 150 + 22 = 172$

$\Rightarrow$ $n = \frac{172}{5} = 34.4$ (It is not a natural number)

Hence, -150 is not a term of given AP.

COMMON ERR(!)R

Sometimes students consider as round off value of n, i.e., 34 is the answer, but it is wrong. Since, 'n' is always a natural number, so we can't round off the value of n.

2. Given, AP: 8, 14, 20, 26, ...

Here, first term $(a) = 8$

and common difference $(d) = 14 - 8 = 6$

Let its nth term will be 72 more than its 41st term.

$\therefore$ $a_n = a_{41} + 72$

$\Rightarrow$ $a + (n-1)d = a + (41-1)d + 72$

$[\because a_n = a + (n-1)d]$

$\Rightarrow$ $(n-1)6 = 40 \times 6 + 72$

$\Rightarrow$ $(n-1)6 = 240 + 72$

$\Rightarrow$ $n - 1 = \frac{312}{6} = 52$

$\therefore$ $n = 53$

So, its 53rd term is the required term.

3. Let a and d be the first term and common difference of an AP. The according to the given condition,

$a_3 + a_7 = 0$

TR!CK

nth term of an AP is: $a_n = a + (n-1)d$

$\therefore$ $[a + (3-1)d] + [a + (7-1)d] = 0$

$\Rightarrow$ $a + 2d + a + 6d = 0$

$\Rightarrow$ $2a + 8d = 0$

$\Rightarrow$ $a + 4d = 0$

$\Rightarrow$ $a + (5-1)d = 0$

$\Rightarrow$ $a_5 = 0$

Hence, 5th term is zero.

4. Let a and d be the first term and common difference of an AP. Then,

$7 \times T_7 = 5 \times T_5$

$\therefore$ $7 \times [a + (7-1)d] = 5 \times [a + (5-1)d]$

$[\because T_n = a + (n-1)d]$

$\Rightarrow$ $7[a + 6d] = 5[a + 4d]$

$\Rightarrow$ $7a - 5a = 20d - 42d$

$\Rightarrow$ $2a = -22d \Rightarrow a = -11d$

$\therefore$ $T_{12} = a + (12-1)d$

$= -11d + 11d = 0$

5. Let a and d be the first term and common difference of an AP. Then,

$a_3 = 5$ and $a_7 = 9$

TR!CK

The nth term of an AP is given by $a_n = a + (n-1)d$

$\Rightarrow$ $a + (3-1)d = 5$ and $a + (7-1)d = 9$

$\Rightarrow$ $a + 2d = 5$ and $a + 6d = 9$

On solving, we get $a = 3$ and $d = 1$

TiP

The series of an AP is a, a + d, a + 2d, a + 3d, ...

$\therefore$ The series of an AP is 3, 3 + 1, 3 + 2, 3 + 3, ...

i.e., 3, 4, 5, 6, ...

6. Let a be the first term and d be the common difference of given AP.

Given, 3rd term of AP, $a_3 = 16$

TR!CK

nth term of AP, $a_n = a + (n-1)d$, where a and d are the first term and common difference respectively.

$\therefore$ $a + (3-1)d = 16$

$\Rightarrow$ $a + 2d = 16$ ...(1)

According to the question,

$a_7 = 12 + a_5 \Rightarrow a_7 - a_5 = 12$

$\Rightarrow$ $[a + (7-1)d] - [a + (5-1)d] = 12$

$\Rightarrow$ $(a + 6d) - (a + 4d) = 12$

$\Rightarrow$ $2d = 12$

$\therefore$ $d = 6$

Put $d = 6$ in eq. (1), we get

$a + 2(6) = 16$

$\Rightarrow$ $a + 12 = 16$

$\therefore$ $a = 4$

Hence, AP will be $a, (a+d), (a+2d), (a+3d), ...$

i.e., $4, (4+6), (4+2\times6), (4+3\times6), ...$

i.e., 4, 10, 16, 22, ...

7. Given AP sequence is $-30, -24, -18, ...$

Let a be the first term and d be the common difference of given AP.

Here $a = -30$, $d = -24 + 30 = 6$

TR!CK

The sum of n terms of an AP is: $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ The sum of 30 terms of an AP is

$$S_{30} = \frac{30}{2}[2\times(-30) + (30-1)6]$$

$$= 15[-60 + 174] = 15 \times 114 = 1710$$

8. Given, $S_n = n(4n+1) = 4n^2 + n$

TR!CK

nth term of an AP, whose sum is S_n, is

$a_n = S_n - S_{n-1}$

$\therefore$ $a_n = 4n^2 + n - [4(n-1)^2 + (n-1)]$

$= 4n^2 + n - [4(n^2 + 1 - 2n) + (n-1)]$

$= 4n^2 + n - [4n^2 + 4 - 8n + n - 1]$

$= 4n^2 + n - [4n^2 - 7n + 3] = 8n - 3$

$\because$ The AP series is $a_1, a_2, a_3, ...$

$\therefore$ The required AP series is $8(1)-3, 8(2)-3, 8(3)-3, ...$

i.e. 5, 13, 21, ...

9. Given, nth term of an AP is

$$a_n = 5 - 2n$$
$$a_1 = 5 - 2(1) = 3$$
$$a_2 = 5 - 2(2) = 1$$
$$a_3 = 5 - 2(3) = -1$$

Here $a = a_1 = 3$

and $d = a_2 - a_1 = 1 - 3 = -2$

$\because$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{20} = \frac{20}{2}[2\times(3) + (20-1)(-2)]$$
$$= 10[6 + 19\times(-2)] = 10[6 - 38] = -320$$

10. Given, $S_n = 3n^2 - 4n$

TiP

If sum of n terms (S_n) of an AP is given, then nth term (a_n) of the AP can be determined by $a_n = S_n - S_{n-1}$ and common difference by $d = a_n - a_{(n-1)}$.

$\therefore$ nth term is given by

$$a_n = S_n - S_{n-1}$$
$$= 3n^2 - 4n - [3(n-1)^2 - 4(n-1)]$$
$$= 3n^2 - 4n - [3(n^2 + 1 - 2n) - 4n + 4]$$
$$= 3n^2 - 4n - 3n^2 - 3 + 6n + 4n - 4$$
$$= 6n - 7$$

and common difference, $d = a_n - a_{(n-1)}$

$$= 6n - 7 - [6(n-1) - 7]$$
$$= 6n - 7 - 6n + 6 + 7 = 6$$

Short Answer Type-II Questions

1. Let 'a' be the first term and 'd' be the common difference of an AP.

Given that, first term $(a) = -14$, last term $(l) = 62$

and fifth term, $a_5 = 2$

$\Rightarrow$ $a + (5-1)d = 2$

[$\because$ nth term of AP is $a_n = a + (n-1)d$]

$\Rightarrow$ $-14 + 4d = 2$

$\Rightarrow$ $4d = 16 \Rightarrow d = 4$

Let n terms are in AP.

$\because$ $l = a + (n-1)d$

$\therefore$ $62 = -14 + (n-1)(4)$

$\Rightarrow$ $4(n-1) = 76 \Rightarrow (n-1) = 19 \Rightarrow n = 20$

So, required number of terms is 20.

2. Let nth term of the AP: 65, 61, 57, 53,...... be the first negative term.

Here, first term $(a) = 65$

and common difference $(d) = 61 - 65 = -4$

$\because$ $a_n = a + (n-1)d$

$\therefore$ $[a + (n-1)d] < 0$

$\Rightarrow$ $65 + (n-1)(-4) < 0$

$\Rightarrow$ $(65 - 4n + 4) < 0$

$\Rightarrow$ $(69 - 4n) < 0$

$\Rightarrow$ $(4n - 69) > 0 \Rightarrow n > \frac{69}{4}$

$\Rightarrow$ $n > 17\frac{1}{4}$

$\therefore$ $n = 18$

So, 18th term is the first negative term.

COMMON ERR(!)R

Sometimes students takes the value of $n = 17$ instead of taken the value of $n = 18$.

3. Let 'a' be the first term and 'd' be the common difference of the given AP.

According to the given condition,

$$a_{24} = 2\times a_{10}$$

$\Rightarrow$ $a + (24-1)d = 2[a + (10-1)d]$

[$\because$ nth term of the AP, $a_n = a + (n-1)d$]

$\Rightarrow$ $a + 23d = 2\times(a + 9d)$

$\Rightarrow$ $a + 23d = 2a + 18d$

$\Rightarrow$ $a = 5d$ ...(1)

Now, $a_{72} = a + (72-1)d = a + 71d$

$= 5d + 71d$ [from eq. (1)]

$\Rightarrow$ $a_{72} = 76d$ ...(2)

and $a_{15} = a + 14d = 5d + 14d$ [from eq. (1)]

$\Rightarrow$ $a_{15} = 19d$ ...(3)

From eqs. (2) and (3), it is clear that

$a_{72} = 4$ times of a_{15} [$\because 76d = 4\times 19d$]

Hence proved.

4. Given that $a_3 = 4$ and $a_9 = -8$

Let 'a' be the first term and 'd' be the common difference of the given AP.

$\because$ nth term of AP, $a_n = a + (n-1)d$

$\therefore$ $a_3 = a + (3-1)d$

$\Rightarrow$ $4 = a + 2d$ ...(1)

and $a_9 = a + (9-1)d$

$\Rightarrow$ $-8 = a + 8d$ ...(2)

On subtracting eq. (1) from eq. (2), we get

$$(a + 8d) - (a + 2d) = -8 - 4$$

$\Rightarrow$ $6d = -12$

$\Rightarrow$ $d = -2$

Put $d = -2$ in eq. (1), we get

$$a + 2\times(-2) = 4$$

$\Rightarrow$ $a = 8$

Let nth term of this AP be zero.

$\because$ $a_n = a + (n-1)d$

$\therefore$ $0 = 8 + (n-1)(-2)$

$\Rightarrow$ $0 = 8 - 2n + 2$

$\Rightarrow$ $2n = 10$

$\Rightarrow$ $n = 5$

Hence, 5th term of this AP is zero.

5. Let 'a' and 'd' be the first term and common difference respectively of an AP.

According to the given condition,

$$a_5 + a_9 = 30$$

$\Rightarrow$ $[a + (5-1)d] + [a + (9-1)d] = 30$

[$\because$ nth term of AP, $a_n = a + (n-1)d$]

$\Rightarrow$ $a + 4d + a + 8d = 30$

$\Rightarrow$ $2a + 12d = 30$

$\Rightarrow$ $a + 6d = 15$ ...(1)

According to the question, $a_{25} = 3\times a_8$

$\Rightarrow \quad a+(25-1)d = 3\times[a+(8-1)d]$

$\Rightarrow \quad a+24d = 3(a+7d)$

$\Rightarrow \quad a+24d = 3a+21d$

$\Rightarrow \quad 2a = 3d \Rightarrow a = \frac{3}{2}d$ ...(2)

On solving eqs. (1) and (2), we get

$\frac{3}{2}d+6d = 15$

$\Rightarrow \quad 15d = 30 \Rightarrow d = 2$

$\therefore \quad a = \frac{3}{2}\times 2 = 3$ [from eq. (2)]

So, required AP is $a, a+d, a+2d, ...$

i.e., $3, 3+2, 3+2\times 2, ...$

i.e., 3, 5, 7, ...

6. Given that tree grows 5 cm or 0.05 m less than preceding year.

TiP

Tree cease growing means the growth of tree becomes zero at some stage.

$\therefore$ The following sequence can be formed:

$1, (1-0.05), (1-2\times 0.05), ..., 0$

i.e., 1, 0.95, 0.90, ..., 0 which is an AP.

Let a and d be the first term and common difference respectively of the given AP.

Here, $a = 1$, $d = 0.95-1 = -0.05$ and $a_n = l = 0$.

$\because \quad l = a_n = a+(n-1)d$

$\therefore \quad 0 = 1+(n-1)(-0.05)$

$\Rightarrow \quad 0.05(n-1) = 1$

$\Rightarrow \quad n-1 = \frac{1}{0.05} = \frac{100}{5} = 20$

$\Rightarrow \quad n = 20+1 = 21$

Hence, in 21 years, tree will have ceased growing.

7. Given AP is $18, 15\frac{1}{2}, 13, ..., -49\frac{1}{2}$

Let a and d be the first term and common difference respectively of the given AP.

Here, $a = 18, d = 15\frac{1}{2}-18 = \frac{31}{2}-18 = \frac{-5}{2}$

and $a_n = l = -49\frac{1}{2} = \frac{-99}{2}$

Let the number of terms be n.

$\because$ nth term of AP, $a_n = a+(n-1)d$

$\Rightarrow \quad \frac{-99}{2} = 18+(n-1)\left(\frac{-5}{2}\right)$

$\Rightarrow \quad -99 = 36-(n-1)(5)$

$\Rightarrow \quad -99 = 36-5n+5$

$\Rightarrow \quad 5n = 99+41 = 140$

$\Rightarrow \quad n = 28$

$\therefore \quad S_{28} = \frac{28}{2}\left[2\times 18+(28-1)\left(\frac{-5}{2}\right)\right]$

$\left[\because \text{sum of first } n \text{ terms of an AP, } S_n = \frac{n}{2}[2a+(n-1)d]\right]$

$= 14\left(36-27\times\frac{5}{2}\right) = 7(72-135) = 7(-63)$

$= -441$

Hence, sum of all given terms is -441.

8. Let a and d be the first term and common difference of an AP respectively.

Given, first term $(a) = 15$

$\therefore \quad S_{15} = 750 \quad \left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$

$\frac{15}{2}[2a+(15-1)d] = 750$

$\Rightarrow \quad 2\times 15+14d = \frac{750\times 2}{15}$

$\Rightarrow \quad 14d = 100-30 = 70$

$\Rightarrow \quad d = 5$

$\therefore \quad a_{20} = a+(20-1)d \quad [\because a_n = a+(n-1)d]$

$= 15+19\times 5$

$= 15+95 = 110$

So, its required 20th term is 110.

9. Let a and d be the first term and common difference respectively of an AP.

Given, $a = 5$, $l = 45$ and $S_n = 400$

$\because$ Sum of first n terms of an AP,

$S_n = \frac{n}{2}(a+l)$

$400 = \frac{n}{2}(5+45) \Rightarrow 400 = \frac{n}{2}(50)$

$\Rightarrow \quad n = \frac{800}{50} \Rightarrow n = 16$

$\because$ nth term of AP, $l = a+(n-1)d$

$\therefore \quad 45 = 5+(16-1)d$

$\Rightarrow \quad 40 = 15d$

$\Rightarrow \quad d = \frac{40}{15} = \frac{8}{3}$

Hence, the number of terms is 16 and the common difference is $\frac{8}{3}$.

10. Let a and d be the first term and common difference respectively of the given AP.

Here, $a = 54$, $d = 51-54 = -3$ and $S_n = 513$.

Let required number of terms be n.

Then, sum of first n terms of the AP,

$\because \quad S_n = \frac{n}{2}[2a+(n-1)d]$

$513 = \frac{n}{2}[2\times 54+(n-1)(-3)]$

$\Rightarrow \quad 513\times 2 = n(108-3n+3)$

$\Rightarrow \quad 1026 = n(111-3n)$

$\Rightarrow \quad 1026 = 111n-3n^2$

$\Rightarrow \quad 3n^2-111n+1026 = 0$

$\Rightarrow \quad n^2-37n+342 = 0$ (divide by 3)

$\Rightarrow \quad n^2-19n-18n+342 = 0$

$\Rightarrow \quad (n-19)-18(n-19) = 0$

$\Rightarrow \quad (n-19)(n-18) = 0$

$\Rightarrow \quad n-19 = 0 \text{ or } n-18 = 0$

$\Rightarrow \quad n = 19 \text{ or } n = 18$

$\therefore$ Sum of 18 terms = Sum of 19 terms

But 19th term is 0.

$[\because a_{19} = 54 + 18 \times (-3) = 54 - 54 = 0]$

Hence, required number of terms is 18.

COMMON ERROR

Some students confused in double answer and make them mistake in writing the answer.

11. Given, first term $(A) = a$, second term $= b$ and last term $(l) = c$

TRICK

nth term of AP, $a_n = l = a + (n-1)d$
where, $a_n = l$ = last term of AP and n is the number of terms.

Now, $l = A + (n-1)d$

$\Rightarrow c = a + (n-1)(b-a)$

($\because$ common difference $d = b - a$)

$\Rightarrow \dfrac{c-a}{b-a} = n-1$

$\Rightarrow n = \dfrac{c-a}{b-a} + 1 = \dfrac{c-a+b-a}{b-a}$

$n = \dfrac{b+c-2a}{b-a}$

$\because$ Sum of first n terms of an AP is $S_n = \dfrac{n}{2}(A+l)$

$\therefore S_n = \dfrac{n}{2}(A+l) = \dfrac{1}{2}\left(\dfrac{b+c-2a}{b-a}\right)(a+c)$

$= \dfrac{(a+c)(b+c-2a)}{2(b-a)}$ **Hence proved.**

Long Answer Type Questions

1. Installments to be paid by a man is 20, 35, 50....
Since, the difference between each consecutive terms is 15 (Constant). So, this sequence forms an AP.
Let a and d be the first term and common difference of an AP.
$\therefore a = 20$ and $d = 35 - 20 = 15$
Suppose the loan is cleared in 'n' months.
$\because$ Sum of the amounts = 3250

$\therefore \dfrac{n}{2}\{2a + (n-1)d\} = 3250$

$\Rightarrow n\{2 \times 20 + (n-1)(15)\} = 6500$

$\Rightarrow n\{40 + 15n - 15\} = 6500$

$\Rightarrow 15n^2 + 25n - 6500 = 0$

$\Rightarrow 3n^2 + 5n - 1300 = 0$

$\Rightarrow 3n^2 + 65n - 60n - 1300 = 0$

(By splitting the middle term)

$\Rightarrow n(3n+65) - 20(3n+65)$

$\Rightarrow (3n+65)(n-20) = 0$

$\Rightarrow n = \dfrac{-65}{3}, 20$

But n cannot be negative because n is a natural number.

$\therefore n = 20.$

Thus, the loan is cleared in 20 months.

2. Given, sum of first 'n' terms of an AP. is. $S_n = 3n^2 + n$
Now, $S_1 = a_1 = a = 3(1)^2 + (1) = 3 + 1 = 4.$
Which is a first term of an AP.
and $S_2 = 3(2)^2 + 2 = 3 \times 4 + 2 = 12 + 2 = 14.$
Now $a_2 = S_2 - S_1 = 14 - 4 = 10.$
$\therefore$ Common difference $(d) = a_2 - a_1 = 10 - 4 = 6$
$\because$ nth term of an AP is,
$a_n = a + (n-1)d$
$\therefore$ 15th term of an AP is,
$a_{15} = a + (15-1)d$
$= 4 + 14 \times 6 = 4 + 84 = 88.$

3. Let the first term and common difference of an AP are 'a' and 'd' respectively.
Given, sum of the first 8 terms of an AP = 100

TiP

Sum of first n terms of an AP is: $S_n = \dfrac{n}{2}[2a + (n-1)d]$.

$\therefore S_8 = 100$

$\Rightarrow \dfrac{8}{2}[2a + (8-1)d] = 100$

$\Rightarrow 2a + 7d = 25$ ...(1)

Also, sum of its first 19 terms is 551.

$\therefore S_{19} = 551$

$\Rightarrow \dfrac{19}{2}[2a + (19-1)d] = 551$

$\Rightarrow a + 9d = 29$ ...(2)

Put the value of 'a' from eq(2), in eq(1).
we get,

$2(29 - 9d) + 7d = 25$

$\Rightarrow 58 - 18d + 7d = 25$

$\Rightarrow -11d = -33 \Rightarrow d = 3$

Now, put the value of 'd' in eq(2), we get

$a + 9 \times 3 = 29 \Rightarrow a = 29 - 27 = 2.$

$\therefore$ Sum of its first 'n' terms is,

$S_n = \dfrac{n}{2}\{2a + (n-1)d\} = \dfrac{n}{2}\{2 \times 2 + (n-1)3\}$

$= \dfrac{n}{2}(4 + 3n - 3) = \dfrac{1}{2}n(3n+1)$

4. Given AP is –12, –9, –6, ..., 21.
Let a and d be the first term and common difference respectively of the AP.
Here, $a = -12$, $d = -9 - (-12) = 3$ and $l = 21 = a_n$
Let n be the number of terms in the given AP.
$\because$ nth term of AP, $l = a_n = a + (n-1)d$

$\therefore 21 = -12 + (n-1)(3)$

$\Rightarrow 21 = -12 + 3n - 3$

$\Rightarrow 21 + 15 = 3n$

$\Rightarrow n = \dfrac{36}{3}$

or $n = 12$

On adding 1 to each term in given AP, new AP so formed is –11, –8, –5, ..., 22.
Here, $a = -11$, $d = -8 - (-11) = 3$, $n = 12$ and $l = 22$

$\therefore$ Sum of n terms of the AP, $S_n = \frac{n}{2}(a + l)$

$\therefore \quad S_n = \frac{12}{2}(-11 + 22) = 6 \times 11 = 66$

Hence, the required number of terms is 12 and sum of all terms of new AP is 66.

5. Let a and d be the first term and common difference respectively of the given AP.

Given, $S_6 = 36$ and $S_{16} = 256$

TR!CK

Sum of first n terms of an AP is: $S_n = \frac{n}{2}[2a + (n-1)d]$.

$\Rightarrow \quad \frac{6}{2}[2a + (6-1)d] = 36$

and $\quad \frac{16}{2}[2a + (16-1)d] = 256$

$\Rightarrow \quad 3\,[2a + 5d] = 36$ and $8\,[2a + 15d] = 256$

$\Rightarrow \quad 2a + 5d = 12 \quad$ ...(1)

and $\quad 2a + 15d = 32 \quad$ ...(2)

On subtracting eq. (1) from eq. (2), we get

$(2a + 15d) - (2a + 5d) = 32 - 12$

$\Rightarrow \quad 10\,d = 20 \Rightarrow d = 2$

Put $d = 2$ in eq. (1), we get

$2a + 5 \times 2 = 12$

$\Rightarrow \quad 2a = 12 - 10 = 2 \Rightarrow a = 1$

Now, $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)2] = \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2}[2n] = n^2$

$\therefore \quad S_{10} = (10)^2 = 100$

Hence, sum of first 10 terms is 100.

6. Let a and d be the first term and common difference of the AP respectively.

According to the question, $S_p = S_q$

TR!CK

Sum of first n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad \frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$

$\Rightarrow \quad p(2a + pd - d) = q(2a + qd - d)$

$\Rightarrow \quad 2ap + p^2d - pd = 2aq + q^2d - qd$

$\Rightarrow \quad 2a(p-q) + d(p^2 - q^2) - d(p-q) = 0$

$\Rightarrow \quad (p-q)\,[2a + d(p+q) - d] = 0$

$\Rightarrow \quad 2a + (p + q - 1)\,d = 0 \; (\because p \neq q)$ ...(1)

Now, sum of first $(p+q)$ terms $= S_{p+q}$

$= \frac{(p+q)}{2}\,[2a + (p + q - 1)d]$

$= \left(\frac{p+q}{2}\right) \times 0 \quad$ [from eq. (1)]

$= 0 \quad$ **Hence proved.**

Chapter Test

Multiple Choice Questions

Q 1. If 7th term and 13th term of an AP are 34 and 64 respectively, then its 18th term is:

a. 89 b. 90 c. 92 d. 94

Q 2. If the sum of n terms of an AP is $3n^2 + n$ and its common difference is 6, then its first term is:

a. 2 b. 4 c. 5 d. 6

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): The common difference of an AP in which $a_{15} - a_{10} = 30$ is 6.

Reason (R): The nth term of the sequence 8, 13, 18, ... is $5n + 3$.

Q 4. Assertion (A): The sum of the series with the nth term $T_n = 4 - 2n$ is -208, when number of terms is 16.

Reason (R): The sum of AP series is determined by $S_n = \frac{n}{2}[2a + (n-1)d]$.

Fill in the Blanks

Q 5. In any arithmetic progression, if each term is increased by 3, then the new progression series is formed ....

Q 6. If the common difference is .... then each term of the AP will be same as the first term of the AP.

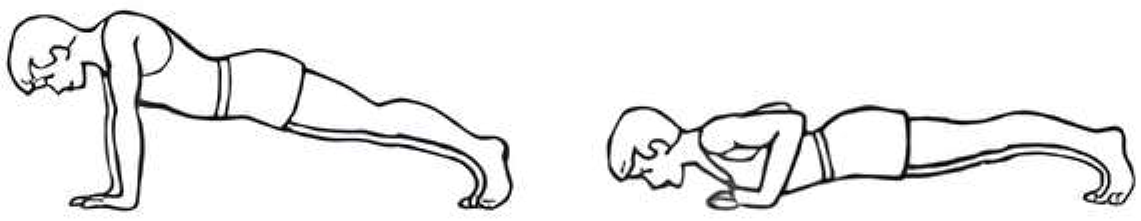
True/False

Q 7. If nth term of an AP is a_n, then the common difference is determined by $d = a_n - a_{n-1}$.

Q 8. A sequence follow certain rule is a progression.

Case Study Based Question

Q 9. Push-ups are a fast and effective exercise for building strength. These are helpful in almost all sports including athletics. While the push-up primarily targets the muscles of the chest, arms and shoulders, support required from other muscles helps in toning up the whole body.



Nitesh wants to participate in the push-up challenge. He can currently make 3000 push-ups in one hour. But he wants to achieve a target of 3900 push-ups in 1 hour for which he practices regularly. With each day of practice, he is able to make 5 more push-ups in one hour as compared to the previous day. If on first day of practice he makes 3000 push-ups and continues to practice regularly till his target is achieved.

Based on the given information, solve the following questions:

(i) Form an AP representing the number of push-ups per day.

(ii) Find the minimum number of days he needs to practice before the day his goal is accomplished.

(iii) If Nitesh wants to achieve a target of 5000 push-ups in 1 hour then find the minimum number of days he need to practice before the day his goal is accomplished.

OR

Find the total number of push-ups performed by Nitesh up to the day his goal is achieved.

Very Short Answer Type Questions

Q 10. Find the 6th term from the end of the AP: 17, 14, 11,......, −40.

Q 11. Find the sum of 20 terms of the AP: 1, 4, 7, 10,..... .

Short Answer Type-I Questions

Q 12. Which term of the arithmetic progression 5, 15, 25,....... will be 130 more than its 31st term?

Q 13. Find the sum of first 30 terms of an AP whose 2nd term is 2 and 7th term is 22.

Short Answer Type-II Questions

Q 14. Find the sum of all odd integers between 2 and 100 divisible by 3.

Q 15. A man repays a loan of ₹3250 by paying ₹20 in the first month and then increases the payment by ₹ 15 every month. How long will it take him to clear the loan?

Long Answer Type Question

Q 16. If the m^{th} term of an AP be $\frac{1}{n}$ and its n^{th} term be $\frac{1}{m}$, then show that its mn^{th} term is 1.